Independent Schools Examinations Board

COMMON ACADEMIC SCHOLARSHIP EXAMINATION

MATHEMATICS

Monday 27 February 2006

Please read this information before the examination starts.

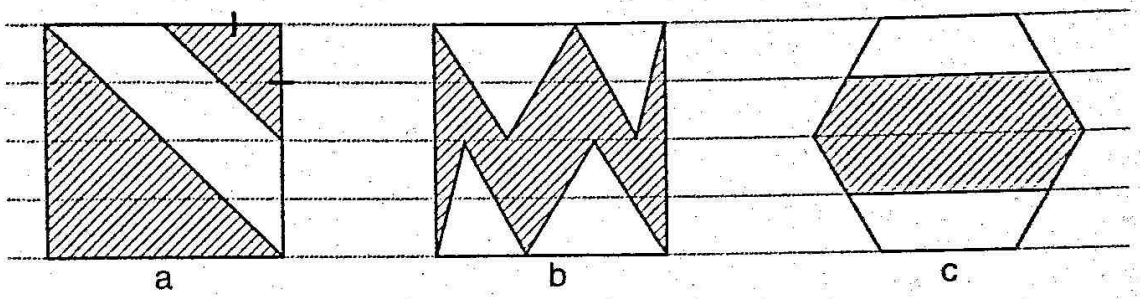
- This examination is 90 minutes long.
- You may **not** use a calculator for this examination.
- You may attempt the questions in any order.
- Show all your working.
- The questions are intended to be increasingly difficult.
- Concentrate on complete solutions to questions rather than fragmentary answers.

1. (a) Evaluate $2\frac{5}{6} \times 3\frac{3}{5} - 4\frac{1}{4}$ (4)

(b) Simplify $\frac{(3x)^2 y}{6x^3 y^2} \times 2xy$ (3)

(c) Given that $6789 \times 6789 = 46\,090\,521$, find 6790×6790 (3)

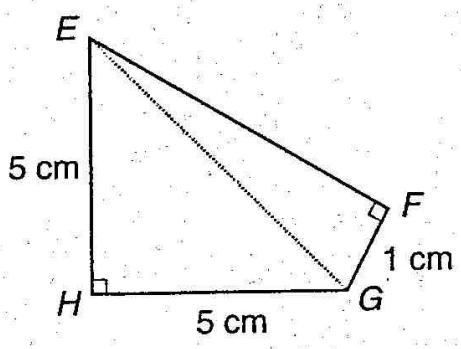
2. Each of these regular polygons is partially shaded.
The horizontal dotted lines are equally spaced.



In each case, find what fraction of the area of the polygon is shaded.

Explain your answers carefully and simplify fractions as much as possible. (3, 3, 4)

3. not to scale



The quadrilateral *EFGH* is shown in the diagram above.

The lengths $GH = HE = 5$ cm, $FG = 1$ cm.
The angle EFG = angle $EHG = 90°$.

(a) Use Pythagoras' Theorem to

(i) show that the length $EG = \sqrt{50}$ (1)

(ii) find the length of *EF*. (2)

A square *ABCD* has the same area as the quadrilateral *EFGH*.

(b) Find the length of the perimeter of the square *ABCD*. (4)

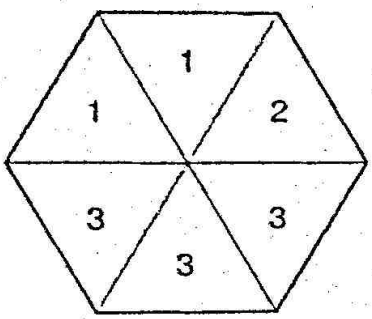
4. (a) Solve the equation $3 - 4(5x - 6) = 7 - 8x$ (3)

(b) (i) Solve the inequality $2(7y - 6) > 5(3y - 4)$ (3)

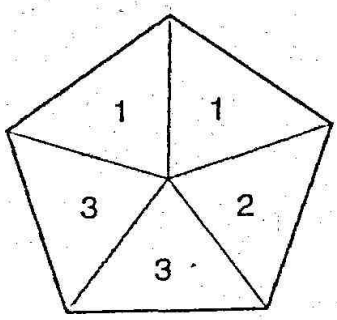
(ii) List all of the prime numbers that satisfy the inequality in part (b)(i). (1)

(c) If $5i + 4j + 8k = 35$ and $j + 2k = 5$, find the value of i. (2)

5.



spinner H



spinner P

A game uses two spinners as shown above.
One is a regular hexagon and the other is a regular pentagon.

(i) What is the probability of scoring a 3 on pentagon spinner P? (1)

Spinner H is spun once and then spinner P is spun once.

(ii) (a) Copy and complete the table below showing all the possible totals from the scores on the two spinners. (3)

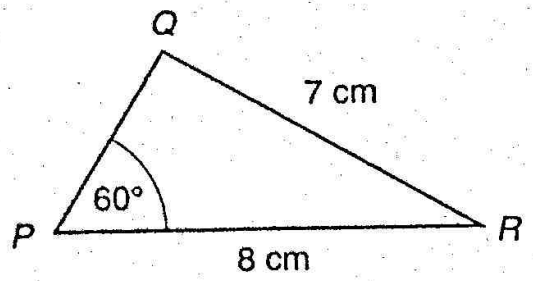
		spinner H					
	+	1	1	2	3	3	3
	1	2	2	3	4	4	4
	1	2				4	
spinner P	2		3		5		5
	3	4				6	
	3			5			

(b) What is the probability of scoring a total greater than 3? (2)

(iii) By making a similar probability table to that in part (ii), or otherwise, find the probability that the difference between the two scores is 1 (3)

(iv) What is the probability that the total of the two scores is twice as much as the difference between them? (2)

6.



not to scale

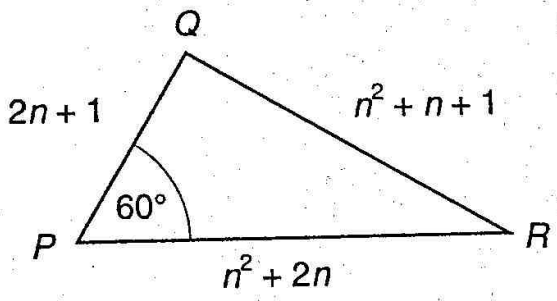
A sketch of the triangle PQR is shown above.

There are two possible values for the length of PQ, both of which are a whole number of centimetres.

(i) Using a ruler, a protractor and a pair of compasses, construct accurately the **two** triangles PQR each showing a different length that PQ can have. (3)

There are many triangles with whole number lengths of sides and a 60° angle between the longest and the shortest side.

One way to find them is to substitute a whole number value for n in the expressions in the diagram below.



not to scale

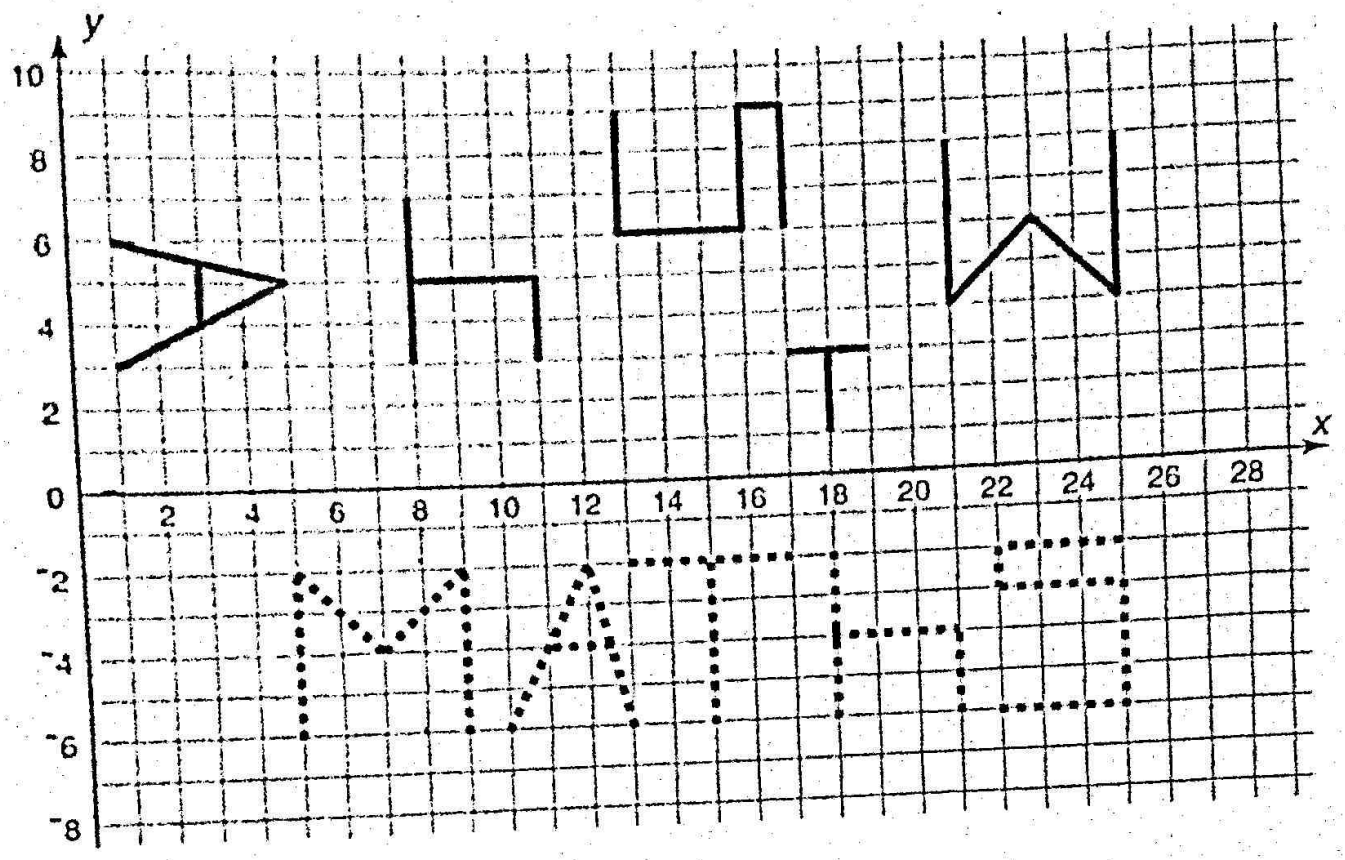
(ii) Calculate the lengths of sides PQ, QR and RP when $n = 7$ (2)

(iii) If $PR = 195$ mm,

(a) write down an equation in terms of n (1)

(b) find n by trial and improvement. (3)

7.



The letters M, A, T, H and S are shown on the graph above.

(a) Describe **fully** a combination of two reflections which together would map the solid M onto the dotted M. (2)

(b) Describe **fully** the **single** transformation which maps

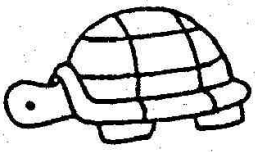
(i) the solid S onto the dotted S (2)

(ii) the solid H onto the dotted H (2)

(iii) the solid T onto the dotted T (2)

(iv) the solid A onto the dotted A. (2)

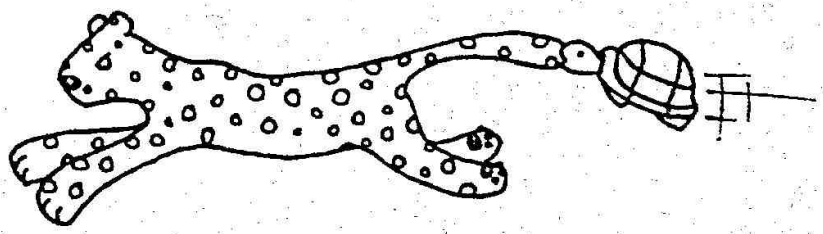
8. A hare and a tortoise ran a race. They started at the same time.
The tortoise started to run along as fast as he could, at a steady 50 cm/s.



(i) How many metres did he run in 20 seconds? (1)

After 20 seconds, a friendly cheetah raced by and the tortoise grabbed his tail.
He was dragged along by the cheetah for 12 seconds at a speed of 12 m/s.

(ii) How far was the tortoise dragged along by the cheetah? (1)



After 12 seconds the tortoise lost his grip and so fell to the ground.

The tortoise stayed still for 6 seconds.

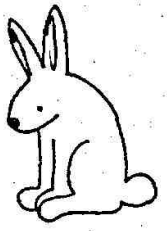
Then he ran on at his previous speed of 50 cm/s and reached the finishing tape t seconds later.

(iii) How many metres did he run in the t seconds? (1)

(iv) Using your answers to parts (i) to (iii), write an expression in terms of t for the total distance that the tortoise travelled. (2)

(v) Write down an expression in terms of t for the total time the tortoise took. (1)

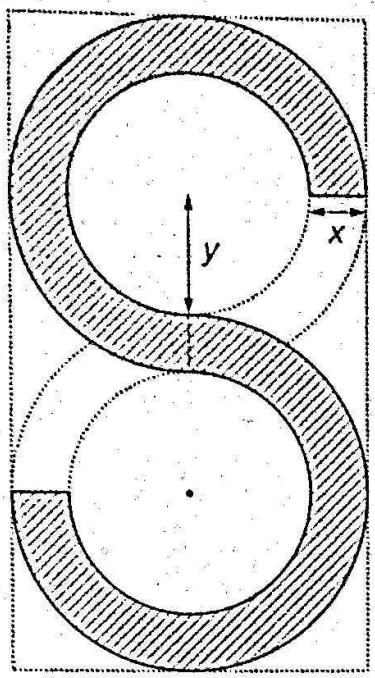
Meanwhile, the hare had been leaping along at a constant speed of 3 m/s and arrived at the finishing tape at exactly the same time as the tortoise.



(vi) Use your answer to (v) to write down an expression in terms of t for the distance the hare had travelled. (2)

(vii) Write down an equation in t, and solve it, to find the total time it took for the hare and the tortoise to run the race. (4)

9.



not to scale

The diagram above shows a design for the letter S.

It is made from two three-quarter circular rings joined at the middle.

The rings have an internal radius of y cm and are x cm wide.

An expression for the total width of the letter S is $(2x + 2y)$ cm.

(i) Write down an expression in terms of x and y for the total height of the letter S. (2)

The letter S has to be 32 cm wide and 60 cm high.

(ii) Using this information, write down two equations in x and y (2)

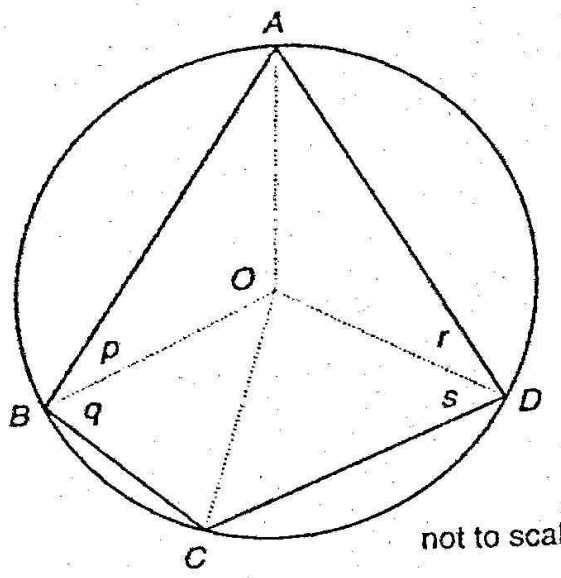
(iii) Solve the equations in (ii). (3)

(iv) Show that the area of the letter S is 168π cm^2. (3)

The letter S is cut from a rectangle of plastic as shown in the diagram above.
The area of the rectangle is $60 \times 32 = 1920$ cm^2.

(v) By making a suitable approximation for π, **estimate** the percentage of the rectangle which is used for the letter S. (2)

10.



not to scale

The quadrilateral *ABCD* in the diagram above fits exactly inside a circle.

(i) Explain why angle *AOB* is equal to $180° - 2p$ (2)

(ii) Find a similar expression for angle *BOC*. (1)

(iii) Form an equation by adding together expressions for the angles

AOB, *BOC*, *COD* and *DOA*

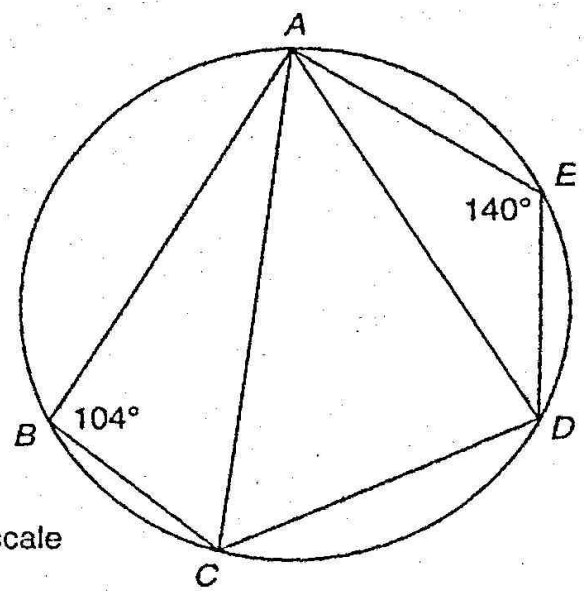
and simplify it to show that $p + q + r + s = 180°$. (3)

(iv) Deduce a connection between the angles *ABC* and *ADC*. (1)

(v) Use this result to find

(a) angle *ADC* (1)

(b) angle *CAD*. (2)



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(Total marks: 100)